

Two-Time-Slot Bidirectional Relaying in Molecular Communication

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Abstract

In this paper, we study the bidirectional/two-way relaying of molecular communication and propose a relaying scheme with two time slots. Compared to the four-time-slot and three-time-slot schemes, the proposed two-time-slot scheme improves the throughput by a significant extent by allowing the end nodes to transmit simultaneously at the very first time slot. In contrast to the existing techniques, the proposed scheme employs a homogeneous molecular communication for bidirectional relaying where all the nodes (i.e., end nodes and relay node) are allowed to operate on the same type of molecule instead of utilizing different types of molecule for different nodes. As a result, this proposal of homogeneous molecular relaying remarkably improves the resource reuse capability. This paper generically characterizes the transmission and detection strategies of the proposed scheme. Moreover, we derive the analytical bit error probabilities for the multiple access and broadcast phases and present the end-to-end bit error probability of the proposed scheme. It's noteworthy that we take it into account the effect of molecular interference in the theoretical derivations. Extensive simulation is carried out, and it is shown that simulation results match very well with the derived theoretical analysis.

Index Terms

Molecular communications, Nano communications, Bidirectional relaying, Diffusion-based communications.

I. INTRODUCTION

Nano-network is a current research area that includes the nanotechnology, biomedical engineering, and communication technology. Nano-network consists of two or more nano-sensors, which are devised in nano-scale to do sensing, computation, and actuation [1]. Nano-sensors can be implanted inside the human body to monitor blood elements, to identify cancer cells, and to deliver drugs with great precision. In order to accomplish these tasks, an efficient and reliable communication (i.e., information exchange) among the nano-sensors is required. Among different communication strategies, diffusion-based molecular communication (DMC) is the most promising communication approach to enable the communication between nano-sensors due to its bio-compatible nature [2]. In DMC, molecules are used to transmit and receive information.

Recently a lot of works have been done on the point-to-point (i.e., single transmitter and single receiver scenario) molecular communication, such as [3–7]. However, only few works have been carried out with considering relay in the molecular communication. Molecular communication for the one-directional relaying is investigated in [2], [8], [9], while [10–12] have investigated molecular communication in the context of bidirectional relaying. In this paper, we focus on the molecular communication for the bidirectional relaying.

In bidirectional relaying, two end nodes exchange their information via a relay node. The traditional bidirectional relaying is generally performed in four time slots, where the two end nodes take turn to send their information to the relay node and the relay node takes two time slots to forward the information from one end node to the other end node. By introducing network coding [13], the required number of time-slot can be reduced to three, where the relay node performs bit-wise XOR operation on the bits received from the end nodes and then broadcasts

the XOR bits to the end nodes in the third time slot. The throughput of the bidirectional relaying can be further improved by carrying out the exchange of information in two time slots [14–16]. In the two-time-slot scheme, the end nodes transmit simultaneously at the first time slot.

A molecular bidirectional relaying with three time slots was shown in [10], [11]. In [12], two-time-slot relaying was shown, however three types of molecules were used to complete the bidirectional relaying.¹ Moreover, the analysis of [12] does not consider the interference from the unwanted molecules.

In this paper, a novel two-time-slot bidirectional relaying for molecular communication is proposed. The contributions of this paper are summarized as follows -

- It proposes a two-time-slot bidirectional relaying model for molecular communication while considering homogeneous molecular relaying. In other words, each transmitting node operates on the same type of molecule rather than using different types of molecule for different transmitting nodes. Thus the proposed scheme greatly enhances resource reuse and efficiency. Moreover, it improves throughput of the bidirectional relaying at a significant degree by allowing the end nodes to transmit simultaneously at the first time slot.
- It broadly characterizes transmission and detection strategies for the proposed scheme.
- This paper derives the analytical bit error probabilities for each time slot and also show the end-to-end bit error probability of the proposed scheme. We also considers the effect of molecular interference in the theoretical derivations. Extensive simulation is carried out to verify the theoretical analysis.

The remainder of this paper is organized as follows. In Section II, we present the system model of the two-time-slot bidirectional relaying in the molecular communication. We analyse the first time slot (multiple access phase) in Section III, while section IV describes the second

¹In [12], although the end nodes transmits simultaneously in the first time slot, the end nodes use two different types of molecules, which is similar to the use of two different frequency for wireless communication. The relay broadcast it's information to the end nodes via third type of molecule. Thus the scheme presented in [12] is analogous to the three-time-slot scheme.

time slot. We present the simulation and analytical results in Section V. Finally, we conclude in Section VI.

II. SYSTEM MODEL

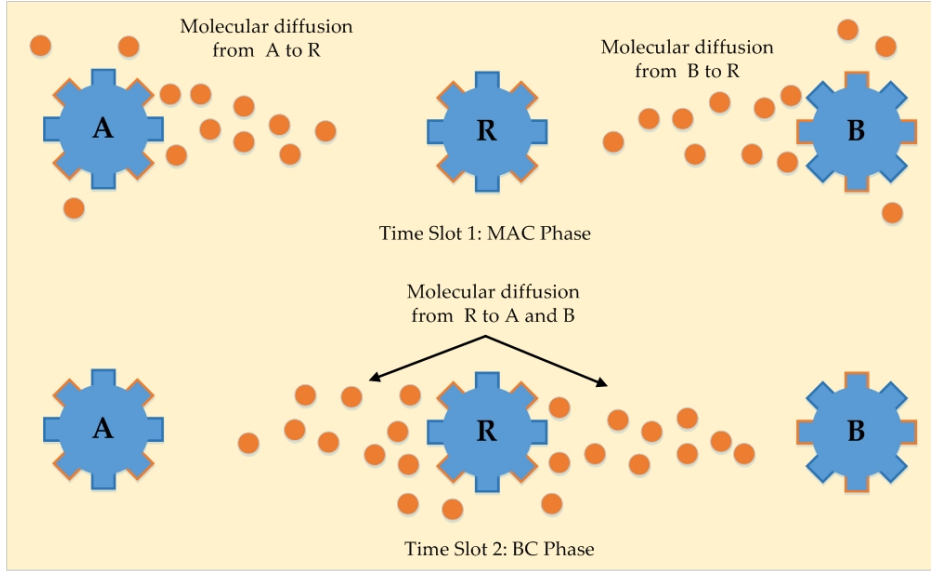


Fig. 1: System model of two-time-slot bidirectional molecular communication, where A and B act as the end nodes and R is the relay node. In the 1st time slot (called the MAC phase), both nodes A and B transmits their information (in the form of molecules) simultaneously and node R receives. The received information is XOR-ed at node R and transmitted to A and B nodes in the 2nd time slot, called the broadcast phase.

A bidirectional relaying model for molecular communication is illustrated in Fig. 1, where two end nodes *A* and *B* exchange their information via a relay node *R*. These nodes transmit their information by emitting molecules, where each molecule propagates independently. We consider a half-duplex communication scenario², where all nodes are assumed to be perfectly synchronized. We also consider that the exchange of the information between two end nodes is accomplished in two time slots.

²In half-duplex communication, nodes does not transmit and receive simultaneously. Depending on the time-slot, a node act as either a transmitter or a receiver.

In the first time slot, known as multiple access (MAC) phase, node A and node B simultaneously transmit their own bit sequences by emitting groups of molecules. For simplicity of notation, throughout the paper, we use a set i as $i \in \{A, B\}$. Let $s_i \in \{0, 1\}$ is required to send from end node i within a bit period t_b . As a modulation scheme, we consider on-off keying [4], hence the end node emits N_i molecules if $s_i = 1$ or the end node keeps silent if $s_i = 0$. Following [5], the probability of a molecule (which was emitted from end node i) absorbed by node R within time t is given by

$$P_{Ri}(d_i, t) = \frac{r_R}{d_i} \operatorname{erfc} \left(\frac{d_i - r_R}{\sqrt{4Dt}} \right), \quad (1)$$

where r_R is the radius of node R , d_i is the distance between end node i and R , and D is the diffusion constant and $\operatorname{erfc}(x)$ is the complementary error function defined as $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-x^2) dx$. In (1), $P_{Ri}(d_i, t)$ is also known as the capture probability. Based on the total number of received molecules from both end nodes, the relay detects the sum of the transmitted bits $s_R = s_A + s_B$, where $s_R \in \{0, 1, 2\}$. Let Y_R be the total number of absorbed molecules at the relay in a certain bit period. Note that Y_R includes molecules from current bit period as well as undesired molecules that arrive from previous bit periods. Based on Y_R , the relay performs following detection:

$$s_R = 2 \quad \text{if } Y_R \geq \tau_2$$

$$s_R = 1 \quad \text{if } \tau_1 < Y_R < \tau_2$$

$$s_R = 0 \quad \text{if } Y_R \leq \tau_1$$

where, τ_1 and τ_2 ($\tau_1 < \tau_2$) are two predetermined thresholds. Since 0 and 2 both indicate that the transmitted bits from the end nodes are same, the relay considers both 0 and 2 as 0. Thus, the relay node essentially detects the X-OR version of the transmitted bits from the end nodes.

We provide the details about the MAC phase in Section III.

In the second time slot, known as broadcast (BC) phase, the relay releases molecules based on the detected X-OR bit sequence. The relay node releases N_R molecules for $s_R = 1$ otherwise the relay does not transmit any molecule. Similar to (1), for a molecule transmitted from the relay, the capture probability of end node i within time t is given by

$$P_i(d_i, t) = \frac{r_i}{d_i} \operatorname{erfc} \left(\frac{d_i - r_i}{\sqrt{4Dt}} \right), \quad (2)$$

where r_i is the radius of node i . Note that the communication scenario in the BC phase is same as the traditional point-to-point molecular communication. At end node i , let Y_i be the total absorbed molecules at BC phase in a specific bit period and $\hat{s}_{R_i} \in \{0, 1\}$ is the detected bit corresponds to s_R . Based on Y_i , end node i detects \hat{s}_{R_i} in the following manner.

$$\begin{aligned} \hat{s}_{R_i} &= 1 && \text{if } Y_i \geq \tau_i \\ \hat{s}_{R_i} &= 0 && \text{if } Y_i < \tau_i \end{aligned}$$

where τ_i is a predefined threshold for end node i . Since each end node knows its own bits, each end node detect the bits of the other end node by performing X-OR between \hat{s}_{R_i} and corresponding own bit. For example, node A detects the bits from node B by $\hat{s}_{B_A} = \hat{s}_{R_A} + s_A$, where \hat{s}_{B_A} is detected at node A corresponds to s_B . The details about the detection in BC phase is given in Section IV.

III. MULTIPLE ACCESS PHASE

Recall that molecules are simultaneously released from end nodes at MAC phase. Due to the molecules sent from end node i , the number of molecules absorbed by the relay during current bit period follows a binomial distribution. Let Y_{Ri_c} denotes the number of received molecules at

the relay within current bit period t_b , provided that molecules were emitted from the end node i . For $s_i = 1$,

$$Y_{Ric} \sim \mathcal{B}(N_i, P_{Ri}(d_i, t_b)), \quad (3)$$

and $Y_{Ric} = 0$ for $s_i = 0$. To enable tractable analysis, the above binomial distribution is approximated by the following Gaussian distribution.

$$Y_{Ric} \sim \mathcal{N}(N_i P_{Ri}(d_i, t_b), N_i P_{Ri}(d_i, t_b) (1 - P_{Ri}(d_i, t_b))). \quad (4)$$

Due to the randomness of the diffusion process, molecules emitted in the previous bit periods may reach at the relay in the current bit period. Thus molecules from the past bit periods cause interference on the reception of the molecules of the current bit period. However, in [4], it was shown that only the molecules that were sent in the last time period has the significant contribution on the interference. Let Y_{Rip} be the number of received molecules at the relay within current bit period that were sent from the end node i during last bit period. Y_{Rip} can be written as

$$\begin{aligned} Y_{Rip} &\sim \mathcal{N}(N_i P_{Ri}(d_i, 2t_b), N_i P_{Ri}(d_i, 2t_b) (1 - P_{Ri}(d_i, 2t_b))) \\ &\quad - \mathcal{N}(N_i P_{Ri}(d_i, t_b), N_i P_{Ri}(d_i, t_b) (1 - P_{Ri}(d_i, t_b))) \\ &= \mathcal{N}(\mu_{Ri2} - \mu_{Ri1}, \sigma_{Ri2}^2 - \sigma_{Ri1}^2), \end{aligned} \quad (5)$$

where $\mu_{Rik} = N_i P_{Ri}(d_i, kt_b)$ and $\sigma_{Rik}^2 = \mu_{Rik} (1 - P_{Ri}(d_i, kt_b))$. Let $Y_{Ri} = Y_{Ric} + Y_{Rip}$ be the total number of molecules (sum of desired and interferer molecules) received in a specific bit period due to the transmitted molecules from end node i . Since Y_{Ri} depends on the current

bit and the last bit of end node i , the distribution of Y_{Ri} can be classified for the four different cases, which are summarized in Table I. In the Table, c_i and p_i denote the current bit and the last bit, respectively at end node i . $D_{Ri}(m)$ represents the shorthand notation of the distribution, where m is the decimal value corresponds to binary number $c_i p_i$.

TABLE I: Distribution of Y_{Ri}

c_i	p_i	Distribution of Y_{Ri}
0	0	$D_{Ri}(0) = 0$
0	1	$D_{Ri}(1) = \mathcal{N}(\mu_{Ri_2} - \mu_{Ri_1}, \sigma_{Ri_2}^2 - \sigma_{Ri_1}^2)$
1	0	$D_{Ri}(2) = \mathcal{N}(\mu_{Ri_1}, \sigma_{Ri_1}^2)$
1	1	$D_{Ri}(3) = \mathcal{N}(\mu_{Ri_2}, \sigma_{Ri_2}^2)$

In a certain bit period, let Y_R be the total absorbed molecules by the relay due to the emitted molecules from both end nodes and thus $Y_R = Y_{RA} + Y_{RB}$. It is clear that Y_R depends on the 16 combinations of input bits of node A and node B . Let $D_R(m)$ be the shorthand notation that represents the distribution of Y_R , where m is the decimal representation of the bit sequence $c_A p_A c_B p_B$. For example, $D_R(9)$ is the distribution of Y_R when the end nodes satisfy $c_A p_A c_B p_B = 1001$. Depending on the combination of bit sequences, all the 16 possible distributions of Y_R can be derived from following equations for $\ell \in \{0, 1, 2, 3\}$:

- $D_R(\ell) = D_{RB}(\ell)$
- $D_R(\ell + 4) = D_{RA}(1) + D_{RB}(\ell)$
- $D_R(\ell + 8) = D_{RA}(2) + D_{RB}(\ell)$
- $D_R(\ell + 12) = D_{RA}(3) + D_{RB}(\ell)$

Let the mean and the variance of $D_R(m)$ are ν_m and ς_m^2 , respectively. Let us define the arrays $\mathbf{u} = [0, 1, 4, 5]$, $\mathbf{v} = [10, 11, 14, 15]$, and $\mathbf{w} = [2, 3, 6, 7, 8, 9, 12, 13]$. For the proposed relaying

scheme, the analytical bit error rate (P_M) at the MAC phase can be derived as:

$$P_M = \frac{1}{16} \left\{ \sum_{p=1}^4 Q\left(\frac{\tau_1 - \nu_{\mathbf{u}(p)}}{\varsigma_{\mathbf{u}(p)}}\right) + \sum_{p=1}^4 \left(1 - Q\left(\frac{\tau_2 - \nu_{\mathbf{v}(p)}}{\varsigma_{\mathbf{v}(p)}}\right)\right) + \sum_{p=1}^8 \left(1 - Q\left(\frac{\tau_1 - \nu_{\mathbf{w}(p)}}{\varsigma_{\mathbf{w}(p)}}\right) + Q\left(\frac{\tau_2 - \nu_{\mathbf{w}(p)}}{\varsigma_{\mathbf{w}(p)}}\right)\right) \right\} \quad (6)$$

where $Q(x)$ is defined by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{x^2}{2}\right) dx$ and $\mathbf{a}(j)$ is the j^{th} element of array $\mathbf{a} \in \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. Recall that, τ_1 and τ_2 are the two predetermined thresholds. Each of the summation terms inside the curly bracket corresponds to the different combination of c_A and c_B . First term results from the case when they (c_A and c_B) both are zeros; second term results from the case when they both are ones and third term results from the case when they are not same. We divide (6) by 16, since each of the possible bit combinations ($c_A p_A c_B p_B$) occurs with same probability.

IV. BROADCAST PHASE

In the broadcast phase, the relay node starts to emit the molecules to the both end nodes depending on the estimated X-OR bit s_R . Due to the transmitted molecules from the relay, let Y_i be the total number of molecules (sum of desired and interferer molecules) received by end node i in a specific bit period. We also denote c_R and p_R as the current bit and last bit, respectively of the relay. Since Y_i depends on the current bit and the last bit of the relay node, the distribution of Y_i depends on four different combinations of c_R and p_R . The distributions of Y_i are summarized in Table II. In the Table, $\mu_{i_k} = N_R P_i(d_i, kt_b)$, $\sigma_{i_k}^2 = \mu_{i_k} (1 - P_i(d_i, kt_b))$ and $D_i(m)$ represents the shorthand notation of the distribution, where m is the decimal value corresponds to binary number $c_R p_R$.

From the distribution of Y_i , we derive the analytical bit error rate (P_{B_i}) at the end node i in

TABLE II: Distribution of Y_i

c_R	p_R	Distribution of Y_i
0	0	$D_i(0) = 0$
0	1	$D_i(1) = \mathcal{N}(\mu_{i_2} - \mu_{i_1}, \sigma_{i_2}^2 - \sigma_{i_1}^2)$
1	0	$D_i(2) = \mathcal{N}(\mu_{i_1}, \sigma_{i_1}^2)$
1	1	$D_i(3) = \mathcal{N}(\mu_{i_2}, \sigma_{i_2}^2)$

the BC phase as:

$$P_{B_i} = \frac{1}{4} \left\{ 2 + Q \left(\frac{\tau_i - \mu_{i_2} + \mu_{i_1}}{\sqrt{\sigma_{i_2}^2 - \sigma_{i_1}^2}} \right) - Q \left(\frac{\tau_i - \mu_{i_2}}{\sigma_{i_2}} \right) - Q \left(\frac{\tau_i - \mu_{i_1}}{\sigma_{i_1}} \right) \right\} \quad (7)$$

Recall that τ_i is a predefined threshold at node i .

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present the simulation and analytical results. To obtain results from simulation and analysis, we set the parameters as $r_A = r_B = r_R = 10\mu\text{m}$, $t_b = 1$ sec, $d_A = d_B = d$, $N_A = N_B$, $\tau_1 = \frac{2\mu_{RA_2} - \mu_{RA_1}}{2}$, $\tau_2 = \frac{2\mu_{RA_2} + \mu_{RA_1}}{2}$, and $\tau_A = \frac{\mu_{A_2}}{2}$. In the simulation, the number of absorbed molecules by the receiver are calculated based on corresponding Gaussian distribution. Note that, we only present the performance of the bidirectional relaying for the information flow from node B to node A. All the presented results are also valid for the information flow from node A to node B due to the symmetry of the end nodes.

A. Comparison between Analytical and Simulation Results

In the following, along with the end-to-end bit error rate (BER), we show the BER in the MAC and BC phase of the bidirectional relaying. For the analytical end-to-end BER, we consider that an end-to-end error occurs if an error event occurs either in the MAC phase or in the BC

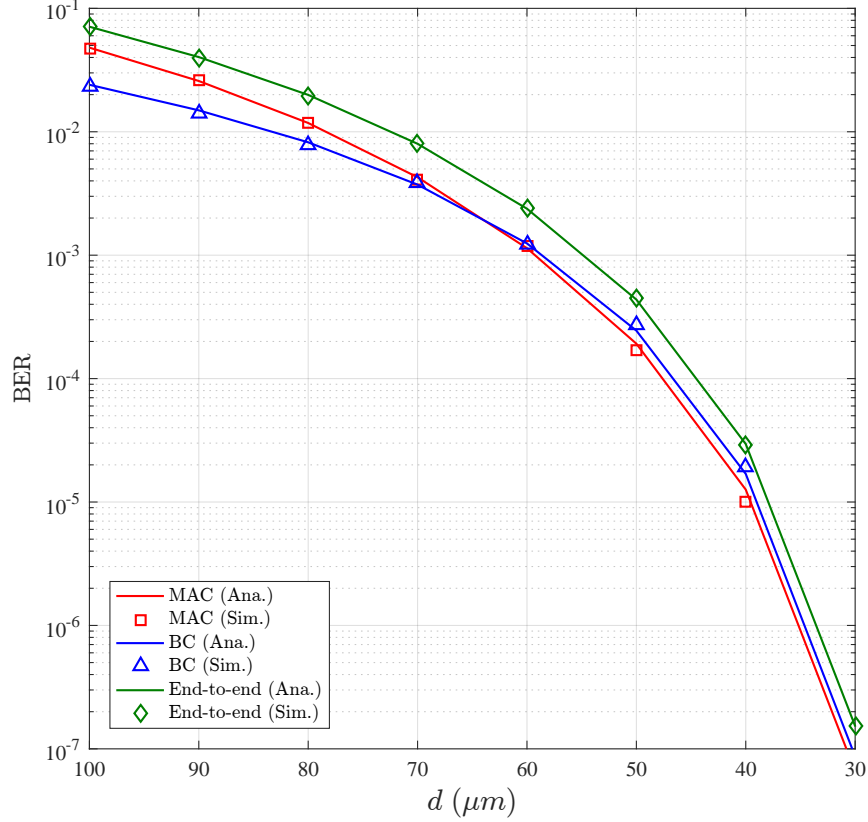


Fig. 2: End-to-end bit error rate (BER) performance along with BER of MAC and BC phases. Both simulation and analytical results are presented. We use $D = 10^4 \mu\text{m}^2/\text{sec}$, $N_A = N_B = 512$ and $N_R = 256$.

phase. Thus, the analytical end-to-end BER (P_A) is given by

$$P_A \approx P_M + P_{B_A}(1 - P_M). \quad (8)$$

Recall that P_M can be calculated from (6) and P_{B_A} can be calculated from (7). In Fig. 2, we present the MAC phase, BC phase, and end-to-end bit error rates. For each case, we show the comparison between BER obtained from the analytical result (*i.e.*, (7), (6), or (8)) and the corresponding BER obtained from simulation result. From Fig. 2, we observe an exact match between the analytical and simulation results, which verifies our analytical derivations. It is also observed that the BER decays by decreasing the distance d , since a small d results in a high

capture probability and vice-versa.

B. Impact of Diffusion Constant and Number of Emitted Molecules

We now show the impact of diffusion constant and number of emitted molecules on the performance of proposed bidirectional relaying. Fig. 3 shows the impact of diffusion constant D on the end-to-end BER, while Fig. 4 shows the impact of number of emitted molecules from the transmitting nodes. It is observed that by increasing the diffusion constant or number of emitted molecules, the overall performance can be improved. As expected, a good match between simulation and analytical results is observed for each case.

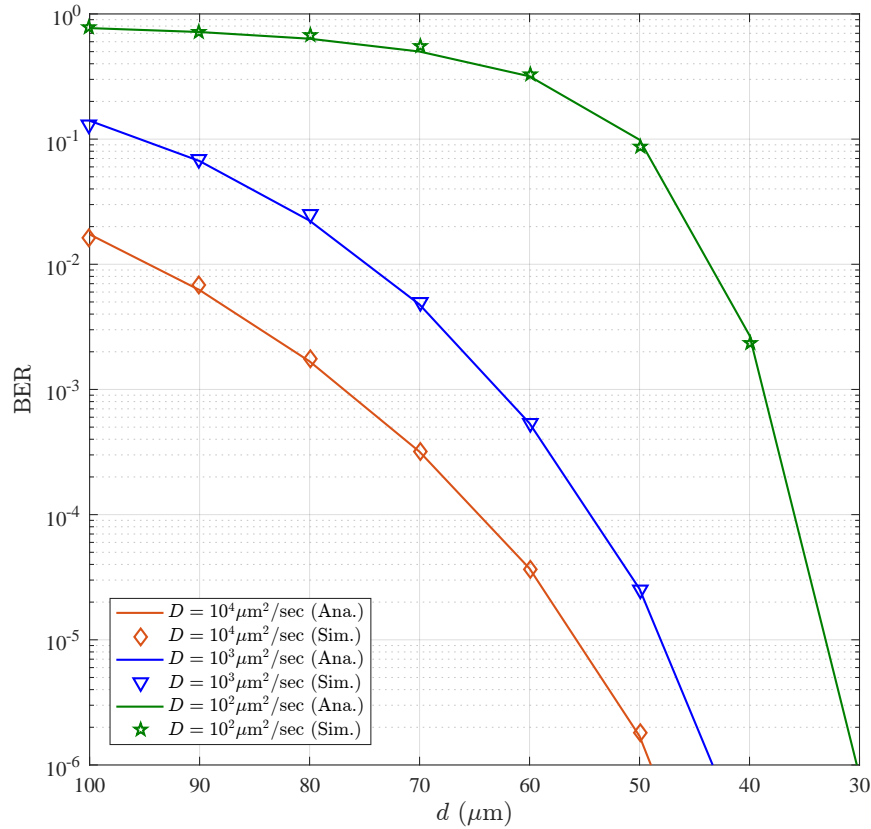


Fig. 3: End-to-end simulation and analytical BER while varying diffusion constant D while we fix $N_A = N_B = N_R = 1024$.

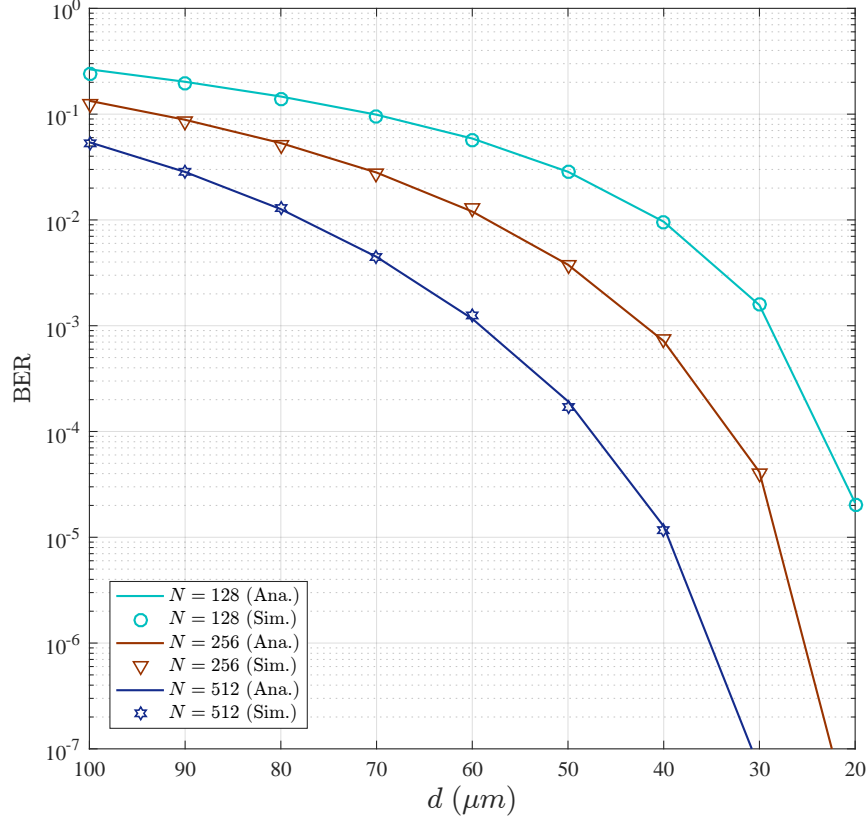


Fig. 4: End-to-end simulation and analytical BER while varying number of emitted molecules $N = N_A = N_B = N_R$. We fix $D = 10^4 \mu m^2/sec$

C. Throughput and Efficiency Comparison with [10–12]

Compared to the traditional bidirectional relaying (*i.e.*, four-time-slot scheme), the proposed two-time-slot scheme increases the throughput by 100%. On the other hand, our proposed scheme increases the throughput by 50% compared to the three-time-slot scheme presented in [10], [11]. Let a single bit can be transmitted in a time slot with a slot duration of 1 second. Then the bit rate (throughput) of four, three, and two time-slot schemes become 15 bits/min, 20 bits/min, and 30 bits/min, respectively. It is worth to mention that in terms of BER performance, the four-time-slot scheme is better than the three-time-slot scheme, while the three-time-slot scheme is better than the two-time-slot scheme.

Due to the use of homogeneous molecules for all the transmitting node, our proposed scheme uses three times less resources compared to the relaying scheme presented in [12], where three types of molecule were used. For example, with three types of molecule, three co-located bidirectional relay networks can be operated using our presented scheme, where each bidirectional relay network can use separate types of molecule and thus, they do not interfere with each other. With this same scenario, the relaying scheme presented in [12] requires nine types of molecule.

VI. CONCLUSION

In this paper, we have proposed a two-time-slot bidirectional relaying scheme. The transmission and detection procedures are presented for the proposed scheme. The proposed two-time-slot scheme is better than traditional scheme in terms of throughput by allowing simultaneous transmission by two end-nodes at the first time slot. Moreover, proposed scheme increases the efficiency (*i.e.*, resource reuse) by employing homogeneous molecules for all transmitting nodes. Analytical bit error probability of different phases are derived for the presented bidirectional relaying scheme. In contrast and as an improvement to earlier works, the presented theoretical analysis considers molecular interference from the past bit periods. Notably, we have observed an exact match between the simulation and analytical results.

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